MTL106 Probability and Stochastic Processes≤

Sem-II 2016-17

TUTORIAL-1 Sigma Field, Independence, Bayes Theorem and General Probability

Solutions

1. By definition of F

{0}, {1}, …, {n}, … all belongs to F

By definition of sigma field countable union of any elements belonging to sigma field must also belong to the sigma field

{0, 2, 4, 6, …} ε F, taking union of all singleton even number sets

But this set is not finite and its complement is set of all odd numbers which is also infinite

Hence it should not be present in the sigma field by the way F is defined and we get a contradiction

This means F is not a sigma field

1. F is the largest sigma field

T = {

Construct a new sigma field F’ = {, T, Tc}

Where TC = { }

Since F is largest sigma field, F’ ϲ F, T ε F and T is an event

P(T) = P(Union over disjoint rectangles into which T can be divided having length dx and width dy)

=

(Using Axiomatic definition of probability has been used to convert required probability into above summation)

=

1. a. Ω = {a,b,c}, F1 = {Φ, {a}, {b,c}, Ω} and F2 = { Φ, {a,b}, {c}, Ω} are two sigma fields on Ω F1 ∪ F2 = { Ω, Φ, {a}, {c}, {a,b}, {b,c} }

U({a}, {c}) does not belong to F, hence F1 ∪ F2 is not a sigma field, Hence FALSE

Consider F1 ⋂ F2 = { Φ, Ω } which is a sigma field

b. P(double headed coin | head comes upon tossing) = P(choosing double headed coin and head comes up upon tossing)/( P(choosing double headed coin and head comes) + P(choosing double tailed coin and head comes) + P(choosing fair coin and head comes) )

=

Hence False

c. A and B be two events with P(A) = ½ and P(Bc) = ¼

if A and B are mutually exclusive events, then P(A) + P(B) ≤ 1 since sum of probabilities has to be less than or equal to 1

But P(A) + P(B) = 0.5 + (1 - 0.25) = 1.25 > 1

Hence A and B cannot be mutually exclusice

FALSE

1. LTAT – Leave on time and arrive on time

LLAT – Leave late and arrive on time

LTAL – Leave on time and arrive late

LLAL – Leave late and arrive late

P(arrive on time | Leave late) = P(LLAT)/( P(LLAT) + P(LLAL) ) = 0.01/(0.01 + 0.03) = ¼

1. Given P(A⋂B) = P(A).P(B)
   1. Whether P(A⋂BC) = P(A).P(BC)

Consider RHS = P(A).(1 – P(B))

= P(A) – P(A).P(B)

= P(A) – p(A⋂B)

= P(A⋂BC)

So A and BC are independent events

* 1. Whether AC and BC are independent events or not

P(AC⋂BC) = (P(A∪B))C (Using DeMorgan’s Law)

= 1 – P(A∪B)

= 1 – (P(A) + P(B) – P(A⋂B))

= 1 – P(A) – P(B) + P(A⋂B)

= 1 – P(A) – P(B) + P(A).P(B)

= (1 – P(A)).(1 – P(B))

= P(AC).P(BC)

AC and BC are independent events

1. No. of ways to in which 40 members of under developed countries can be seated = 40!

No. of ways in which 4 developed countries members can be seated between these 40 members so that none of these 4 members are adjacent to each other =

Number of favorable ways =

Total number of ways in which all 44 members can be seated = 44!

Required probability =

1. Let A be the event that subsystem A fails

Let B be the event that subsystem B fails

P(A) = 0.20

P(A ⋂ B) = 0.15

P(AC ⋂ B) = 0.15

P(AC ⋂ B) = P(B) – P(A ⋂ B) = 0.15

P(B) = P(A ⋂ B) + 0.15 = 0.30

1. P(A / B) = = =
2. P(A⋂BC / A∪B) = P(A⋂BC)/P(A∪B) (we get this numerator since A⋂BC is contained in A∪B)

=

= (0.20 – 0.15)/(0.20 + 0.30 – 0.15)

= ¼

1. Let x be the distance between a and l

Let y be the distance between a and m

0 < x < b –a and 0 < y < b-a

We need to find P(x < y) =

= = ½

1. P(first black / second red) = P(first black and second red)/(P(first black and second red) + P(first and second both red))

=

1. Let A be the altitude of the triangle

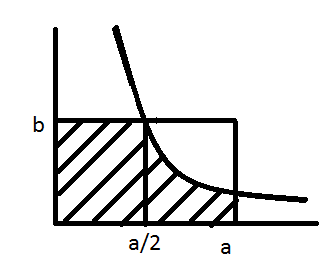
And B be the base

0 < A < a

0 < B < b

Area = ½ . AB < ab/4

AB < ab/2 implying that B < ab/2A



Required probability = shaded area/total area

= (log2 + 1)/2